Modeling Magnetized Plasma

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Abstract: We have developed a simplified model for a symmetric plasma problem in cylindrical coordinates without ion-atom collisions under the action of an axial magnetic field. It neglects the terms which are due to the transformation from Cartesian to cylindrical coordinates when all plasma characteristics except for the azimuthal ion velocity are computed. A comparison of the solutions obtained from the full and the simplified models are in good agreement with the corresponding data.

A symmetric plasma problem under the action of an axial magnetic field in cylindrical coordinates without ion-atom collisions can be described by the following set of normalized equations:

Continuity Equation

$$\frac{d}{d\xi}(\xi y u) = \xi S y$$

Momentum Equations for Ions

$$u\frac{du}{d\xi} + Su + \frac{1}{y}\frac{dy}{d\xi} + b_i(u_{e\theta} - u_{i\theta}) - \underline{\frac{u_{i\theta}^2}{\xi}} = 0$$

$$u\frac{du_{i\theta}}{d\xi} + \underline{\frac{uu_{i\theta}}{\xi}} + Su_{i\theta} + b_i u = 0$$

Momentum Equation for Electrons

$$\frac{d\eta}{d\xi} + \frac{1}{y}\frac{dy}{d\xi} + b_i u_{e\theta} = 0$$

$$u\frac{du_{e\theta}}{d\xi} + \underline{\frac{uu_{e\theta}}{\xi}} + Su_{e\theta} - \frac{m_i}{m_e}b_i u + \alpha_e u_{e\theta} = 0$$

with the initial condition at the plasma center $\xi=0$:

$$y(0)=1 \qquad u(0)=u_{i\theta}(0)=u_{e\theta}(0)=0 \qquad \eta(0)=0$$

At the plasma boundary $\xi=1$: $u(1)=1$

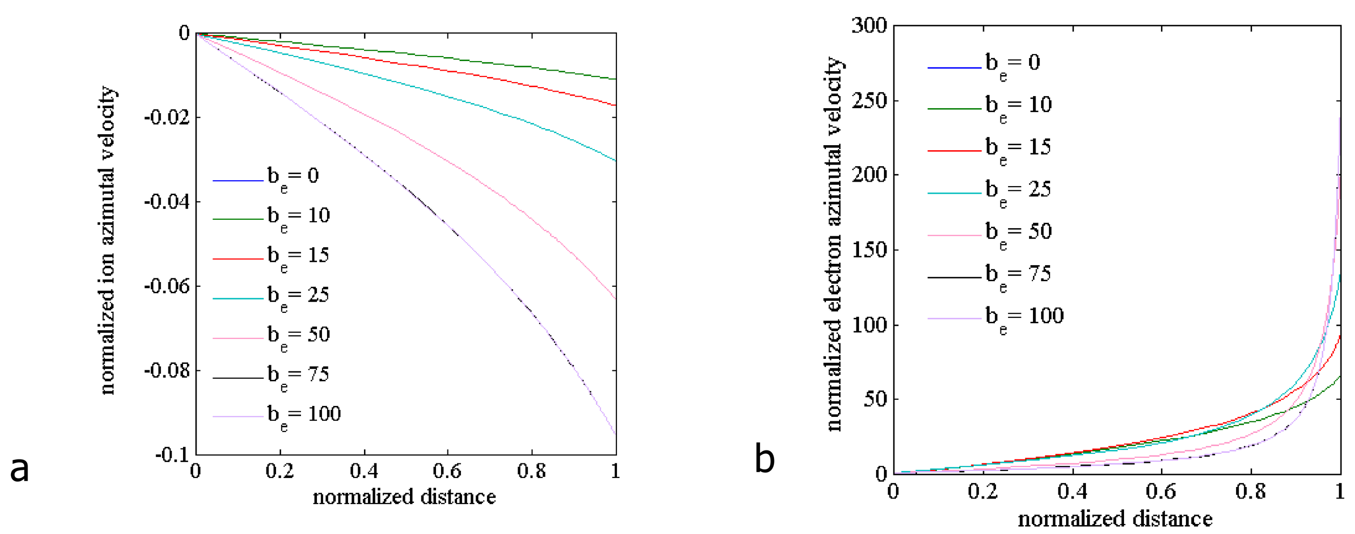

Fig. 1. Normalized (a) ion and (b) electron azimuthal velocities for various magnitudes of the magnetic field.

The underlined terms in the equations are due to the transformation of the coordinate axes from Cartesian to cylindrical.

As seen in Fig. 1, the ion and electron azimuthal velocities vanish when there is no magnetic field present (b_e=b_i=0). Thus, the underlined terms can be omitted, although the system remains in cylindrical coordinates. When there is an axial magnetic field present can the underlined terms also be neglected?

To answer this question, we consider the following simplified system of equations :

Simplified System

$$\frac{d}{d\xi}(\xi y u) = \xi S y$$

$$u\frac{du}{d\xi} + Su + \frac{1}{y}\frac{dy}{d\xi} + b_i(u_{e\theta} - u_{i\theta}) = 0$$

$$u\frac{du_{i\theta}}{d\xi} + \underline{\frac{uu_{i\theta}}{\xi}} + Su_{i\theta} + b_i u = 0$$

$$\frac{d\eta}{d\xi} + \frac{1}{y}\frac{dy}{d\xi} + b_i u_{e\theta} = 0$$

$$u\frac{du_{e\theta}}{d\xi} + Su_{e\theta} - \frac{m_i}{m_e}b_i u + \alpha_e u_{e\theta} = 0$$

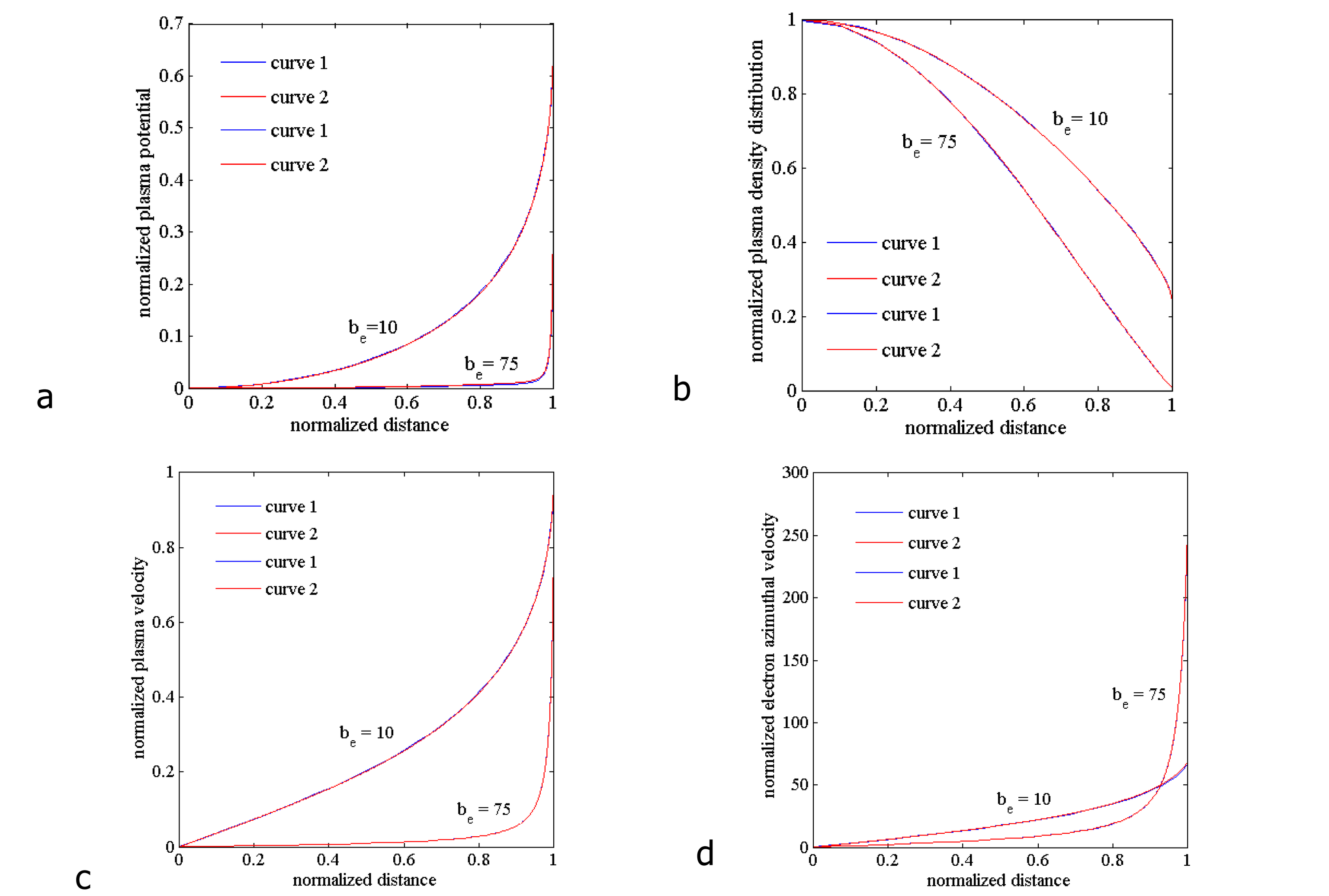

Fig. 2. Comparison of the (a) normalized potential, (b) normalized plasma density distribution, (c) normalized plasma radial velocity, (d) normalized electron azimuthal velocity, obtained by solving the full and simplified plasma models. Curve 1 corresponds the solution of the full equations, while Curve 2 represents the solution of the simplified equations.

We have solved both models, the full and the simplified, for argon gas (m_i/m_e=7344), α_e=30 and $0 \le b_e \le 100$. The results of our computations are shown in Fig. 2 for a low magnetic fields (b_e=10) and for a high magnetic field (b_e=75). In both case, the corresponding curves practically coincide.

As the magnetic field is increased ($b_e \ge 100$), slight discrepancies only in the potential and electric field are observed, but they remain within the experimental range, as illustrated in Fig. 3.

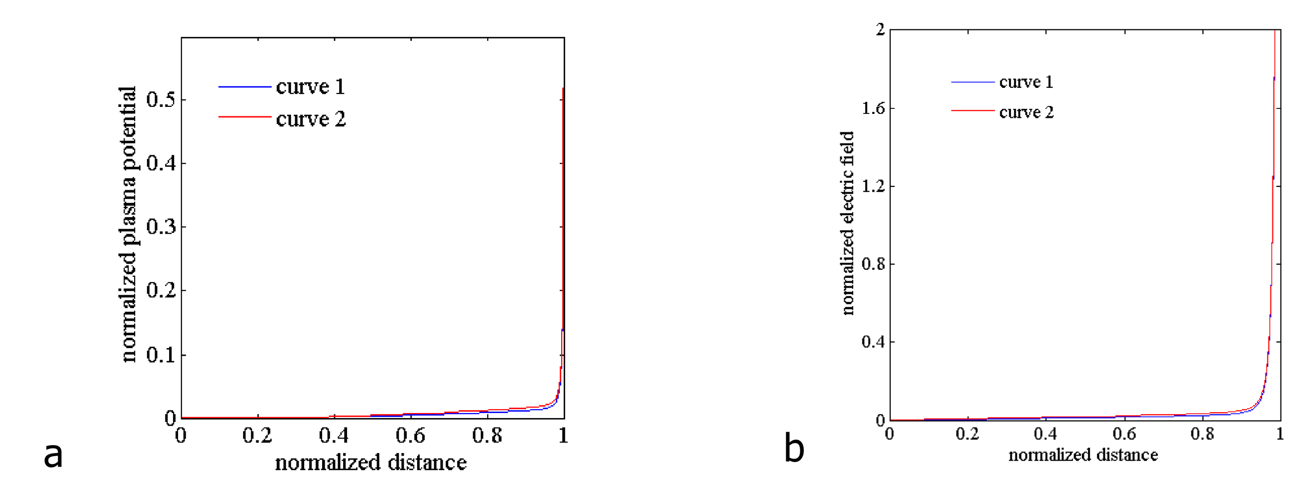

Fig. 3. Comparison of the (a) normalized potential and (b) normalized electric field obtained by solving the full and simplified plasma models for b_e=100. . Curve 1 corresponds the solution of the full equations, while Curve 2 represents the solution of the simplified equations.

Notation:

- ξ normalized radial distance
- y normalized plasma density distribution
- u normalized plasma radial velocity
- $u_{i\theta}$ normalized ion azimuthal velocity
- $u_{e\theta}$ normalized electron azimuthal velocity
- η normalized potential
- S normalized ionization frequency

Parameters:

- R radius of the cylinder
- ρ_i ion cyclotron radius
- m_i ion mass
- m_e electron mass
- b_i ion magnetic field parameter, $b_i = R/\rho_i$
- b_e electron magnetic field parameter, $b_e = b_i (m_i/m_e)^{1/2}$
- α_e electron-atom collision parameter

Summary: When a plasma model is transformed from Cartesian to cylindrical coordinates terms (underlined in the equations) appear in the normalized equations as the result of the transformation itself. We have demonstrated through this computational experiment that those terms are unnecessary in the calculations of magnetized and non-magnetized plasma characteristics, with the exception in the calculation of the ion azimuthal velocity. This was calculated through an iterative algorithm we have developed to find the ionization frequency using the Runge-Kutta method in MATLAB.

References: N. Sternberg, V. Godyak and D. Hoffman, Magnetic field effects on gas discharge plasmas, Phys. of Plasmas, 13, 063511 (2006) .